1. (a) By equation (6.4), \[ \frac{\Delta r}{\Delta d} = -\frac{t}{\ell} = -\frac{50}{0.20} = -250. \]

(b) Moving five miles closer to the city centre will cause bid rent to rise by 1,250. Since bid rent at \( d = 10 \) is 5,000, bid rent at \( d = 5 \) must be 6,250 per acre or 1,250 per lot.

(c) Someone living at \( d = 10 \) pays 1,000 per month in rent and 500 per month in commuting cost, so the total is 1,500 per month. Someone living at \( d = 5 \) pays 1,250 per month in rent and 250 per month in commuting cost, so the total is again 1,500 per month. This illustrates the concept of a spatial equilibrium: rents change along the bid rent function so that identical households get the same level of utility regardless of where they locate. Rents change along the bid rent function so that no one has an incentive to move.

(d) The bid rent function is a line with a slope of -250, so \( r(d) = r(0) - \frac{50}{0.20}d \), where \( r(0) \) is the vertical intercept, or the rent at the city centre. Then, \( r(10) = 5,000 \) implies \( 5,000 = r(0) - \frac{50}{0.20}(10) \), so \( r(0) = 7,500 \), and the bid rent function is \( r(d) = 7,500 - \frac{50}{0.20}d \) (any other point on the curve will allow you to solve for \( r(0) \)). The diagram should look like Figure 6.7.

2. (a) The total demand for land is 25,000 acres, or approximately 39.1 square miles. The total supply of land within a circle of radius \( b \) is \( \pi b^2 \). If everyone is to have a place to live, we must have \( 39.1 = \pi b^2 \), which implies \( b \) is equal to approximately 3.5 miles.

(b) Since the residential and agricultural bid rent functions intersect at the border, we must have \( r(3.5) = 1,000 \) per acre.

(c) The slope of the bid rent function is \( -\frac{250}{0.25} = -1,000 \), so the rent on land at the city centre must be \( 1,000 + 3.5(1,000) = 4,500 \) per acre.
(d) The illustration should look similar to Figure 6.8.

3. If consumers can substitute other goods for land, then, as the price of land rises, they will consume less land and more of other goods. This implies that lot sizes will be smaller where land prices are higher. Since lot size is in the denominator of the slope of the bid rent function, substitution or flexibility will cause the slope to decrease (in absolute value) as lot size increases. This implies that the bid rent function will be convex: steeper near the city centre and flatter near the city boundary.
4. (a) Profit is \( P(d) = 2,000 - 500 - 0.2(1,000)d - R(d) \), and so setting profit equal to zero implies a commercial bid rent function of \( R(d) = 1,500 - 200d \).

(b) For a firm located at \( d = 1 \), expenditures on land equal \( 1,500 - 200 = 1300 \) and shipping costs equal 200 for a total of 1,500. For a firm located at \( d = 2 \), expenditures on land equal \( 1,500 - 400 = 1,100 \) and shipping costs equal 400 for a total of 1,500. This illustrates the concept of a spatial equilibrium: rents change along the bid rent function so that identical firms earn the same level of profit (in this case zero) regardless of where they locate. Rents change along the bid rent function so that no one has an incentive to move.

(c) The location of the border satisfies \( 1,500 - 200d = 1,000 - 100d \), so \( 100d = 500 \) or \( d = 5 \). The area of the commercial ring is thus \( 3.14(25) = 78.5 \) square miles or 50,240 acres. Since each firm occupies one acre, this is also the number of firms in the commercial district.

(d) (i) The firm bid rent function would shift upward, and the size of the commercial district would increase.

(ii) The firm bid rent function would become steeper, and the size of the commercial district would decrease.